Laboratory modeling of atmospheric flow phenomena: Mountain waves

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Abstract—Laboratory simulations in water tanks provide an attractive alternative to full-scale field experiments, moreover, they can be utilized to benchmark analytical and numerical calculations. Here we discuss the possibilities and limitations of modeling large scale atmospheric flow in laboratory. As a case study, we describe experiments on quasi two-dimensional mountain wave formation behind obstacles towed through a linearly stratified fluid. Differences between measured wave fields and predictions of linear theories indicate that nonlinear effects are significant in our parameter range. Experiments with a double bell-shaped obstacle revealed that average wave amplitudes at high enough flow velocities are systematically lower than those produced by an isolated obstacle. We attribute this anomaly to the dominance of essential nonlinearities such as strong wave dispersion and resonance effects.

Key-words: dynamical similarity, stratified flow, mountain waves, laboratory experiments, wave superposition.

1. Introduction: Dynamical similarity

Situations, where an exact solution for the equations of motion can be given, are exceptional in fluid dynamics. Therefore, alternative methods, mostly numerical procedures and laboratory experiments have been developed for elucidating flows that can not be rigorously calculated. A useful starting point for this development is the following question: under what condition do similar flow patterns occur in two geometrically similar arrangements? When such conditions exist and they can be fulfilled, the two flows are said to be dynamically similar (Tritton, 1988).
Let us recall the simplest governing equation in non-dimensional form, which is relevant at environmental flow phenomena (constant density and incompressibility are assumed):

\[ \frac{d\tilde{u}}{dt} = -\frac{1}{Ro} \tilde{n} \times \tilde{u} - \frac{1}{Ro} \nabla p - \frac{1}{Fr^2} n + \frac{1}{Re} \Delta \tilde{u}, \quad (1) \]

where \( \tilde{u} \) is the dimensionless velocity, \( \tilde{n} \) is a vertical unit vector, \( p \) is the dimensionless pressure, and there are three non-dimensional combinations of characteristic parameters, the Rossby number, Froude number, and Reynolds number expressed as follows

\[ Ro = \frac{U}{f_0 L}, \quad Fr = \frac{U}{\sqrt{gL}}, \quad Re = \frac{UL}{v}, \quad (2) \]

where \( U \) and \( L \) as typical velocity and length scales, \( f_0 \) as the Coriolis parameter (~10^{-4} 1/s at Hungary), and \( v \) is the kinematic viscosity. The boundary conditions can be similarly converted by means of the non-dimensional variables. It is easy to see that if the Rossby, Froude and Reynolds numbers are the same for two situations, then the solutions are the same and the same flow patterns occur.

It is not evident that the largest scale flow phenomena in the atmosphere and oceans can be successfully modeled in laboratory tanks. This is mainly because the characteristic sizes are enormously dissimilar. The difference between a cyclone (\( L \approx 1000 \) km) and its laboratory model (\( L \approx 10 \) cm) is 7 orders of magnitude, while the velocity scales are more similar. The widely different length scales usually make impossible to reproduce atmospheric or oceanic Reynolds numbers too, especially when the medium (air or water at ambient temperature) is the same in the experiments. Large Reynolds numbers are attained in special supercooled Helium pressure chambers, but rotation can not be imposed without difficulties for such an equipment. It should be also noted that exact dynamical similarity cannot be fulfilled. In the context of ship model testing, for example, shrinking of \( L \) requires an increase of \( U \) in order to keep the Reynolds number, but a reduction of \( U \) is necessary for a constant Froude number. Both constraints can not be satisfied simultaneously.

Fortunately, laboratory modeling is not hopeless. First of all, viscosity can be neglected in most of the interesting situations, apart from narrow boundary layers or direct turbulence studies. Therefore relatively low Reynolds numbers (~10^3) in the experiments are acceptable. Secondly, the viscous drag and wave drag are usually not coupled for an obstacle surrounded by moving fluid, thus the effects of changing Froude number can be investigated separately. Thirdly, geometric downscaling does not yield to unrealistic speeds. As an example, Fig. 1 shows a realization of the classical experiment by Fultz et al. (1959) to demonstrate baroclinic instability in a rotating tank. When we cool the center by placing ice into the middlemost chamber, thermal convection starts in the second segment by a characteristic velocity of a few mm/s. Thus a rotation speed regulated in the range...
of 1-60 rpm gives a coverage of Rossby number interval $10^{-3}$-$10^{-1}$, which is highly relevant in geophysical contexts. (We note that other non-dimensional control parameters fit to the very experiment even better, the so-called thermal Rossby number and Taylor number, for details see e.g. Phillips (1963)).

Fig. 1. Thermal convection in a rotating tank of three concentric cylinders. The central chamber is filled with ice, the outermost segment can be heated, the convecting medium in between is water at room temperature ($h = 5$ cm). The distance between the copper walls is $L = 10.5$ cm, the actual rotating speed is 10 rpm. (The two inclined stripes at the plexiglass bottom are part of the construction.)

The second example elaborated in the next section is mountain waves in a stratified atmosphere (see Fig. 2). The similarity criteria which must be met in small-scale towing tank experiments have been reviewed by Baines and Manins (1989). The relevant physical quantities are the towing velocity $U$, the fluid depth $H$, the uniform buoyancy (or Brunt-Vaisala) frequency $N = \sqrt{-(g/\rho)\partial\rho/\partial z}$, the maximum obstacle height $h$, the obstacle half-width $w$, and the kinematic viscosity $v$. These quantities give the dimensionless numbers $U/NH$, $U/N2w$, and $U/Nh$ related to wave propagation, wave drag, and horizontal perturbation velocity, respectively. By taking typical towing speeds $U = 1\text{-}15$ cm/s, measured buoyancy frequency values in the experiments $N_{exp} = 1.09\text{-}1.55$ 1/s and for the atmosphere $N_{atm} = 0.03\text{-}0.04$ 1/s, the matching of dimensionless numbers indicates that our setup simulates atmospheric flow up to a level of 5-10 km at an obstacle height of 600-800 m for uniform wind speed in the range of 10-70 m/s (see Table 1). As it is already mentioned, the Reynolds numbers in the experiments ($Re_{exp} \approx 10^2\text{-}10^3$) are much smaller than in the atmosphere ($Re_{atm} \approx 10^6\text{-}10^9$). Another essential difference is the compressibility of the atmosphere. The air density at the tropopause is $\sim40\%$ of the surface value, while the corresponding difference in the towing tank filled with salt water can not be larger than a few percents. Furthermore, upward-propagating gravity waves radiate to infinity in the atmosphere, whereas, in a
towing tank, they can be reflected from the fluid surface behaving as a rigid lid. Related experimental tests (Baines, 1977) indicate that the upper boundary has a substantial effect at relative large obstacle heights \( h/H > 0.15 \).

![Wave field behind a moving (from right to left) bell-shaped obstacle in a linearly stratified fluid.](image)

**Fig. 2.** Top: Wave field behind a moving (from right to left) bell-shaped obstacle in a linearly stratified fluid (salt solution periodically colored by food dye). \( H = 32 \text{ cm}, h = 2 \text{ cm}, \) the towing speed is \( 2.06 \text{ cm/s}, N = 1.26 \text{ 1/s}. \) Bottom: Wave field reconstruction by digital image processing.

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<td>650 m</td>
<td>1 km</td>
<td>0.03 1/s</td>
<td>0.07</td>
<td>0.33</td>
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*Table 1.* Typical parameter values for the laboratory experiments and meteorological data measured in the winter of 1997/98 around mountain Pilis.

Many other demonstrations and experiments on stratified and rotating fluids are summarized (in Hungarian) by Gyüre et al. (2006).
2. Case study: Mountain waves in the laboratory

2.1. Motivation

Internal gravity waves of topographic origin are ubiquitous in the stably stratified atmosphere and oceans (Nappo, 2002). Upward propagating lee-waves at around stationary lenticular clouds in mountain ranges were discovered by German glider pilots in 1933 (Whelan, 2000). Since that time, wave gliding has widely exercised and became very popular, especially at high altitude attempts: while thermals rarely rise higher than 2-3 km, the most energetic mountain waves can penetrate deeply in the stratosphere. The conditions of exploitable wave generation are a stable and smooth density stratification, proper orography, and steady wind field with minimal shear. Such conditions are fulfilled mainly at two locations in Hungary, in case of appropriate meteorological circumstances: the mountain Kékes (the highest isolated peak in the country), and mountain Pilis (with a maximal elevation of 756 m only). The later area is depicted in Fig. 3.

Fig. 3. Topographic settings at around the mountain Pilis (47.7°N, 18.8°E). The wind direction supporting mountain waves is indicated by a heavy arrow. The surface reconstruction on the right shows an area of 90x80 km², white frame locate the map borders.

The surface reconstruction in Fig. 3 illustrates well that the topography around Pilis is quite complex, it is formed by a series of ridges. This has motivated our experiments on wave field behind a double bell-shaped obstacle realizing the simplest case beyond an isolated, symmetric bump.

Uniform flow over two-dimensional obstacles represents the simplest related model system which has been studied extensively since the pioneering works of Lyra (1943), Queney (1948), Long (1953, 1955) and Scorcer (1978). It is not immediately obvious that such a simplified description might have any environmental relevance. Fig. 4 illustrates, however, that in some cases the lower atmosphere possesses nearly ideal physical properties: almost uniform wind from a
constant direction in a stable stratification with an approximately constant Brunt-Vaisala frequency. Indeed, during the winter of 1997/98, wave-gliders succeeded to ascend very high at the lee side of the mountain Pilis. Seven of the flights exceeded the height of 6 km, the best of them was 8250 m (Kassai, 1998).

2.2. Experiments and results

Experiments were performed in a plexiglass tank (length 240 cm, width 8.7 cm, height 40 cm) filled with uniformly stratified fluid to a depth of 32-37 cm by the standard double-bucket method (Fortuin, 1960). Food dye was periodically added up to the mixture at the filler nozzle resulting in a horizontally layered coloring. Disturbances are generated by towing an obstacle with a tense wire along the bottom of the tank from one end to the other.

In an earlier work, we concentrated on asymmetric obstacles and concluded that the shape of the lee side is the determining factor in wave generation (Gyüre and Jánosi, 2003). Preliminary experiments with a double bell-shaped obstacle with a peak-to-peak separation of 20 cm revealed that wave superposition is highly nontrivial, therefore, we extended our analysis in this direction. Methodology, wave field characterization, initial transients, etc., are described in detail by Gyüre and Jánosi (2003).
Fig. 5 shows a comparison of wave patterns behind a single and double obstacles, the other parameters (filling height, stratification and towing speed) are the same. The waves always move with the same velocity as the obstacle, thus they manifest standing patterns in a co-moving frame of reference. A direct visual check clearly indicates that the pattern behind the double bumps is not a simple superposition of two wave fields produced by an isolated hill. It is also apparent that a quantitative characterization of such waves is quite complicated. Their shape is far from being a simple harmonic function, wave breaking, formation of rotors, and various distortions are prevalent. An approximate description is attempted by extracting average wave amplitudes (vertical distance between consecutive minima and maxima) and average wave lengths (horizontal separation between consecutive extremes belonging to the same streakline). An example is shown in Fig. 6.

Fig. 5. Top: Wave field behind a single bell-shaped obstacle towed from right to left in a linearly stratified salt solution, $U = 1.53 \text{ cm/s}, N = 1.26 \text{ l/s}, H = 37.5 \text{ cm}, h = 2.0 \text{ cm}, w = 2.6 \text{ cm},$ Gaussian form. Bottom: Wave field behind a double bell-shaped obstacle fabricated from two identical Gaussian bumps (the same as above) joined with a separation of 12.0 cm, $U = 1.58 \text{ cm/s},$ the other parameters are the same as above.

Theories predict that the flow is linear if $U/Nh$ is sufficiently large, nonlinearities (steepening, wave breaking and mixing, columnar disturbances, etc.) become increasingly important as $U/Nh$ drops beneath unity (Baines, 1995). This is clearly indicated in Fig. 6b, where the scatter of amplitude values makes any conclusion very difficult. Data points above each other belong to the same towing speed but to different heights, unfortunately the dependence is not monotonous. Some indication of constructive interference with the double obstacle might be
present in the range $0.55 < U/Nh < 0.75$ (larger amplitudes than for the isolated obstacle). In the linear regime, a breakdown of wave amplitudes is more pronounced for the double obstacle (Fig. 6a).

![Fig. 6. (a) Normalized amplitudes as a function of dimensionless towing velocity for the single (heavy squares) and double (empty circles) obstacles. Solid lines only guide the eyes. (b) The same as (a), zoomed to the bottom left corner (nonlinear regime, see text).](image)

3. Closing remarks

Atmospheric flows over mountains and hills contain a rich variety of phenomena, many of which occur on scales which are unresolved by numerical weather prediction models. These phenomena include turbulent wakes, the occurrence of flow separation on the lee slope and, when the flow is stably-stratified, gravity-wave generation, severe downslope wind storms and lee vortex shedding. A better understanding of these flows will allow significant improvements to local weather forecasting, especially for aviation.

We have illustrated that laboratory experiments on orographic flows in stratified fluids can provide useful information complementing field observations and numerical modeling. Such experiments are strongly idealized and mimic simple situations, still they are able to reveal the limitations of model computations and help to find correct interpretations of measured meteorological data.
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References


